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The APCI model

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Overview

1. Background
2. APCI model
3. Parameter estimates
4. Smoothing
5. Conclusions

1 Background

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- CMI released new projection spreadsheet.
- Calibration is done by new APCI model.
- See Continuous Mortality Investigation (2017).

2 APCI model

2 APCI model

$$\log m_{x,y} = \alpha_x + \beta_x(y - \bar{y}) + \kappa_y + \gamma_{y-x} \tag{1}$$

2 Related models for $\log m_{x,y}$

$$\begin{aligned}
\text{Age-Period} &: \alpha_x + \kappa_y && (2)\\
\text{APC} &: \alpha_x + \kappa_y + \gamma_{y-x} && (3)\\
\text{APCI} &: \alpha_x + \beta_x(y - \bar{y}) + \kappa_y + \gamma_{y-x} && (4)\\
\text{Lee-Carter} &: \alpha_x + \beta_x\kappa_y && (5)
\end{aligned}$$

2 APCI model

APCI model can be viewed as either:

- An APC model with added Lee-Carter-like β_x term, or
- A Lee-Carter-like model with added γ_{y-x} cohort term.

2 Constraints required

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All of these models require identifiability constraints:

$$\text{Age-Period}: \sum \kappa_y = 0 \tag{6}$$

$$\text{Lee-Carter}: \sum \kappa_y = 0, \sum \beta_x = 1 \tag{7}$$

$$\text{APC}: \sum \kappa_y = 0, \sum \gamma_c = 0, \sum c\gamma_c = 0 \tag{8}$$

(9)

2 Constraints required

APCI model requires five identifiability constraints:

$$\sum \kappa_y = 0 \tag{10}$$

$$\sum (y - y_1)\kappa_y = 0 \tag{11}$$

$$\sum \gamma_c = 0 \tag{12}$$

$$\sum c\gamma_c = 0 \tag{13}$$

$$\sum c^2\gamma_c = 0 \tag{14}$$

2 APCI model

- APCI model requires more constraints than other models.
- Constraints impact the parameter estimates in important ways.

2 Not all cohorts are equal

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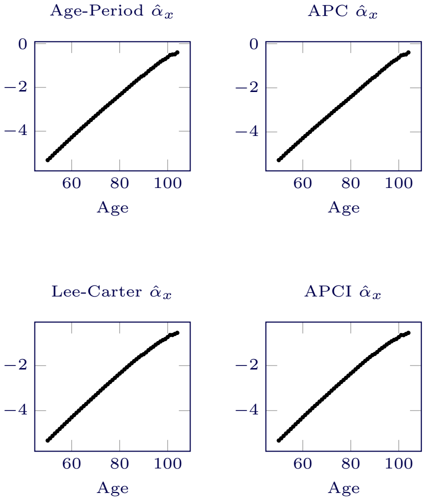
- Continuous Mortality Investigation (2017) uses (for example) $\sum \gamma_c = 0$.
- Cohorts with one observation get same weight as cohorts with thirty observations.
- Cairns et al. (2009) weights according to number of observations, i.e. $\sum w_c \gamma_c = 0$.
- Cairns et al. (2009) approach preferable, so used from now on.

3 Parameter estimates

Parameter estimates $\hat{\alpha}_x$ for four unsmoothed models.

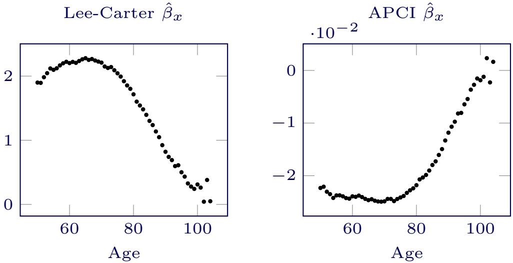


3 α_x

The α_x parameters play the same role across all four models.

3 β_x

Parameter estimates $\hat{\beta}_x$ for Lee-Carter and APCI models (both unsmoothed).

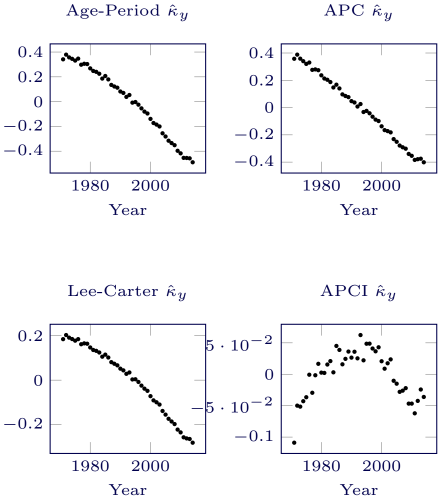


3 β_x

Despite the apparent difference, a switch in sign shows that the β_x parameters play analogous roles in the Lee-Carter and APCI models, namely an age-related modulation of the response in mortality to the time index.

3 κ_y

Parameter estimates $\hat{\kappa}_y$ for four unsmoothed models.

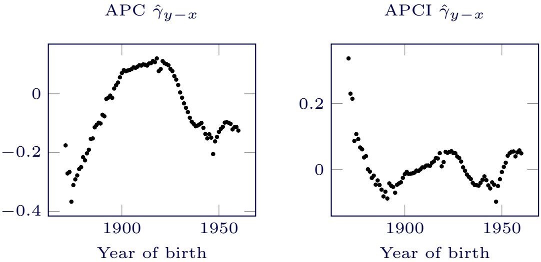


3 κ_y

- While κ_y plays a similar role in the Age-Period, APC and Lee-Carter models, it plays a very different role in the APCI model.
- APCI $\hat{\kappa}_y$ values are at least one order of magnitude smaller than in the other models, and with less of a clear trend pattern.
- In the APCI model κ_y is much more of a residual or left-over term, whose values are therefore strongly influenced by structural decisions made elsewhere in the model.

3 γ_{y-x}

Parameter estimates $\hat{\gamma}_{y-x}$ for APC and APCI models (both unsmoothed).



3 γ_{y-x}

- The γ_{y-x} values play analogous roles in the APC and APCI models...
- ...yet the values taken and the shapes displayed are very different.
- If values and shapes are so different, what does this say about using the APCI γ_{y-x} values to represent cohort effects in the CMI spreadsheet?

4 Smoothing

4 To smooth or not to smooth?

- Continuous Mortality Investigation (2017) smoothes all parameters.
- However, only α_x and β_x exhibit regular behaviour.
- Does it make sense to smooth κ_y and γ_{y-x}?

4 To smooth or not to smooth?

- CMI's smoothing parameter for κ_y is S_κ.
- Value is set subjectively.
- What is the impact of smoothing κ_y?

4 Impact of smoothing κ_y

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life expectancies are [...] very sensitive to the choice made for S_κ, with the impact varying across the age range. At ages above 45, changing S_κ by 1 has a greater impact than changing the long-term rate by 0.5%."

Continuous Mortality Investigation (2016, page 42)

See also https://www.longevitas.co.uk/site/informationmatrix/signalornoise.html

4 Impact of smoothing κ_y

- S_κ has a large impact in part because κ_y is a residual or noise-like term.
- If κ_y is like noise, should one smooth it at all?
- Should one not project κ_y stochastically?

5 Conclusions

5 Conclusions

- APCI model is interesting addition to model pantheon.
- APCI model shares features with APC and Lee-Carter models.
- Smoothing α_x and β_x seems sensible.
- Smoothing κ_y and γ_{y-x} is not sensible.
- Why not turn the APCI model into a full stochastic model?

References I

Cairns, A. J. G., D. Blake, K. Dowd, G. D. Coughlan, D. Epstein, A. Ong, and I. Balevich (2009). A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. *North American Actuarial Journal 13(1)*, 1–35.

Continuous Mortality Investigation (2016). *CMI Mortality Projections Model consultation — technical paper*. Working Paper 91.

Continuous Mortality Investigation (2017). *CMI Mortality Projections Model: Methods*. Working Paper 98.

References II

More on longevity risk at www.longevitas.co.uk